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This is the final progress report for AFOSR Contract #F49620-03-1-0184, funded during the period April-December 2003. It summarizes the work done towards development of a new approach to vibration control of flexible structures. The main goal of this research is to improve control authority in structural systems through methodical stiffness variation, specifically, using end forces, such as follower forces.

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FINAL REPORT

IMPROVED CONTROL AUTHORITY IN FLEXIBLE STRUCTURES USING STIFFNESS VARIATION

AFOSR Contract #F49620-03-1-0184

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This is the final progress report for AFOSR Contract #F49620-03-1-0184, funded during the period April-December 2003. It summarizes the work done towards development of a new approach to vibration control of flexible structures. The main goal of this research is to improve control authority in structural systems through methodical stiffness variation, specifically, using end forces, such as follower forces.

Background

Structural control remains a topic of relevance due to the high performance demands of certain structures, such as the revolutionary lightweight and active mirror system proposed for space-based imaging [14]. Due to their extremely lightweight design, the active mirror system and other highly flexible structures have a large number of flexible modes that need to be actively controlled for satisfactory performance. Although the traditional approach of modal control can be used to suppress vibration in these flexible structures, their limitations pose significant challenges in control design tasks that require a high level of precision and accuracy.

There exists a large volume of literature on the control of structures, and review papers (Balas [3], Hyland, et al. [9], Nurre, et al. [13], for example) provide an excellent summary of the development in the field. One of the fundamental problems in modal control of large flexible structures is spillover (Balas [1], [2], Sesak and Coradetti [15]), which stems from reduced-order models designed for control simplification. Although we do not propose to directly address the problem of spillover, we are developing a control methodology based on structural stiffness variation that has the potential to essentially sidestep the spillover problem. Another disadvantage of traditional approaches based on modal control is the need for estimation and control of each significant mode of the system; this requires a large number of actuators and sensors and a high-dimensional state space model. Our control methodology, in comparison, will provide control authority over the significant flexible modes using a relatively small number of sensors and actuators and a low-dimensional state space model. This is accomplished by variation of the stiffness of the structure that results in modal energy redistribution, along with a

control design that dissipates energy from only a few specific modes in each stiffness state. To the best of our knowledge, such an approach has not been proposed for control of flexible structures.

While there are many ways to vary the stiffness of a structure, we focus primarily on the application of end forces, such as follower forces, which lends itself well to our control design. The dynamics of beams and other structural elements have been widely investigated under the application of follower forces (Bolotin [4], Herrmann [5], Higuchi and Dowell [7], Hodges, et al. [8], for example), however, structural control with follower forces has not been previously considered.

Technical Approach

The basic idea of control design based on stiffness variation is illustrated in Fig.1. The figure shows the control system block diagram of a dynamical system that lends itself to control authority enhancement through stiffness variation. The overall control system is comprised of a stabilizing controller and a stiffness variation law. The stiffness variation law changes the structural stiffness of the plant and modifies the control law to account for variation in the structural parameters. The mechanism for parameter variation involves additional hardware, but such hardware additions are justified by the benefits of control authority improvement. Similar to the control law, the stiffness variation law is based upon feedback, but its time scale of operation will be different from that of the controller.

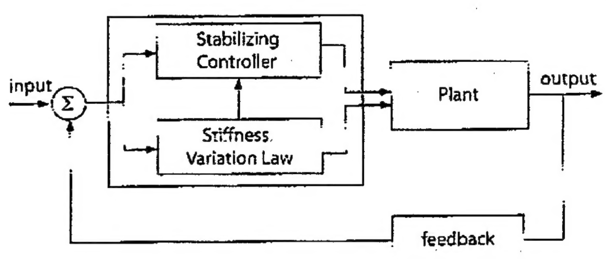

Figure 1: Basic concept of control using stiffness variation.

The basic idea is to modify the structural characteristics "on the fly" in a systematic manner that allows one to access many structural modes with a minimal number of sensors and actuators. This is accomplished by varying structural parameters such that the mode shapes of the structure are modified. Since the modes in the different parameter regimes do not coincide, removal of energy from a single (or a few) mode(s) in each of the parameter regimes in fact removes energy from many modes, due to the modal disparity. Thus, by using a conventional modal control in conjunction with varying modes, one can reduce vibration energy across a wide range of modes.

The test problem considered in this seed project is the straight cantilever beam with an end force applied at the free end. The end force may be a follower force or a combination

of a follower force and an axial force. An end force changes the modal characteristics of the beam, essentially by changing its effective stiffness. By stepping the level of the end force between two values, one can apply modal control in each of the two states, which allows one to access energy across all modes of the beam (at least in theory).

Under discontinuous switching, the closed-loop system will behave as a hybrid dynamical system, consisting of a family of continuous-time sub-systems and the rules governing the switching between them. Most of the work in this area has focused on system stability [10] and the results indicate that slow switching with sufficient "dwell time" can guarantee stability when the individual sub-systems are stable [6], [11], [12]. While stability of the sub-systems should be guaranteed by control design, a loss in performance may result from a conservative dwell time. If there is excessive loss of performance, one can consider the design of more fast switching sequences that will maintain stability.

In order to predict the performance of the proposed approach, one must be able to quantify, or at least estimate, the modal disparity of a given structure, the effectiveness of the modal controller, the energy input associated with the changes in stiffness, and how these interact with damping, noise, unmodeled dynamics.

Modal Disparity: The Proof of Concept

Consider the cantilever (clamped-free) beam of length L and uniform cross section, shown in Fig.2, which is subjected to a follower force P. If E refers to Young's module, I the second area moment, ρ the mass density and A the cross section area, then the equation of motion, for small deflections and assumption of Euler-Bernoulli beam, is [4]

Fig.2: A Cantilever beam with a follower force

$$\begin{cases} EI\dfrac{\partial^4 y}{\partial x^4} + \rho A\dfrac{\partial^2 y}{\partial t^2} + P\dfrac{\partial^2 y}{\partial^2 x} = 0 \\ y(0,t) = y'(0,t) = y''(L,t) = y'''(L,t) = 0 \end{cases} \tag{1}$$

where, $y(x,t)$ is lateral deflection of the beam. Since the boundary conditions of Eq.(1) are the same as the boundary conditions of a cantilever beam without follower force, Eq.(1) can be discretized using normalized mode shapes of the free-end cantilever beam, or "assumed mode shapes", as follows,

$$y(x,t) = \{\delta\}^T \{\phi\} = \sum_{i=1}^{n} \delta_i(t)\phi_i(x) \tag{2}$$

where $\phi_i(x)$ are assumed mode shapes, n is the number of modes, and $\delta_i(t)$ is the modal displacement associated with the i[th] assumed mode shape. By plugging Eq.(2) into Eq.(1) and projecting on to the modes, yields

$$\{\ddot{\delta}\} + \left([K]_{nxn} + P[C]_{nxn}\right)\{\delta\} = 0 \tag{3}$$

where $[K]$ is a diagonal positive definite matrix and $[C]$ is a non-symmetric matrix. The follower force, P, is considered to be a positive quantity.

In this equation, modal displacements are coupled because of the assumed mode shapes. To obtain mode shapes of the beam with end force, we consider following transformation

$$\{\delta\} = [T]\{z\} \tag{4}$$

where columns of matrix $[T]$ are formed using eigenvectors of $[K]_{nxn} + P[C]_{nxn}$. Substitution of Eq.(4) into Eq.(3) results in

$$\{\ddot{z}\} + [\Omega]\{z\} = 0 \tag{5}$$

where $[\Omega] = T^{-1}([K] + P[C])\,T$. In fact, $[\Omega]$ is a diagonal matrix consisting of the square of the natural frequencies of the beam for $P < P_{flutter}$. Now, plugging the expression for $\{\delta\}$ from Eq.(4) into Eq.(2), yields

$$y(x,t) = \{z\}^T [T]^T \{\phi\} = \sum_{i=1}^{N} z_i(t)\psi_i(x) \tag{6}$$

in which $\{\psi\} = [T]^T\{\phi\}$ are the "true" mode shapes of the beam.

To see the difference between mode shapes of a cantilever beam with and without follower force, the first four mode shapes of a cantilever beam for two cases of $P = 0$, and $P = 75\,N$ are depicted in Fig.3. These modes-shapes correspond to an aluminum beam. *1.25* meters long, *3* millimeters thick, and *50* millimeters wide. For this beam the first two natural frequencies of this beam are about *9.8* and *62 rad/sec* and the flutter force is about *100 N*.

Exploiting Modal Disparity in Control Design

As seen in Fig.3, end-force can significantly change the mode shapes of a beam (especially the lower modes). Our goal is to exploit these changes in modal frequencies and shapes in control design with the objective of vibration suppression. In order to demonstrate the modal disparity idea, we consider an idealized static problem wherein the follower force switched between two values, p_1 and p_2, and the fundamental modal

components are repeatedly removed from the system after each switch. We also assume that energy added/removed due to work done by follower force is negligible. We will show that such a strategy removes energy from the beam, including higher modes, in systematic manner, and requires only to be able to control the fundamental mode corresponding to each of two values of p_1 and p_2. For the calculations we denote the attendant normalized mode shapes for p_1 and p_2 as $\phi_j(x)$ and $\psi_j(x)$, respectively. If one starts with a beam deflection $y_0(x,t)$ and an end load of p_1, then we can write

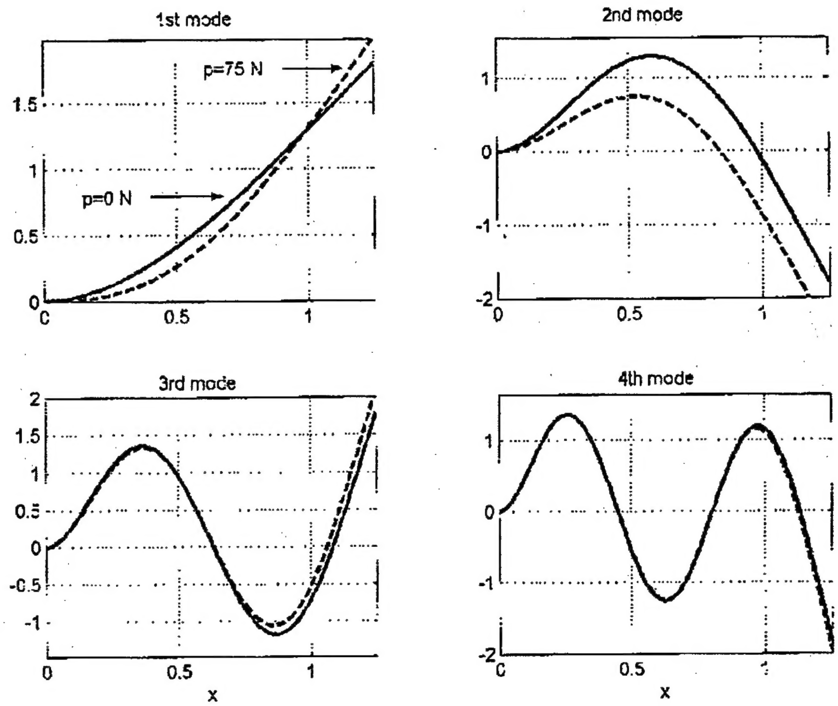

Fig.3: Mode shapes corresponding to $p = 0$ and $p = 75N$

$$y_0(x,t) = \sum_{j=1}^{N} \delta_j(t)\phi_j(x) \tag{7}$$

Assuming one can remove the first mode using a simple controller, the resulting shape is given by

$$y_1(x,t) = y_0(x,t) - \delta_1(t)\phi_1(x) = \sum_{j=2}^{N} \delta_j(t)\phi_j(x) \tag{8}$$

At this point the end-load is switched to p_2 and the shape is now conveniently expressed as

$$y_1(x,t) = \sum_{j=1}^{N} \beta_j(t)\psi_j(x) \tag{9}$$

It is assumed that the first mode is again removed while maintaining end force p_2. This results in the shape

$$y_2(x,t) = \sum_{j=2}^{N} \beta_j(t)\psi_j(x) \tag{10}$$

The end load is then switched back to p_1 where the shape can then be expressed by

$$y_2(x,t) = \sum_{j=1}^{N} \gamma_j(t)\phi_j(x) \tag{11}$$

This completes one cycle of the process, and one is interested in how the new modal coefficients, the γ_j's are related to the originals, the δ_j's. This is conveniently described by a linear mapping

$$\Gamma = [M]\Delta \tag{12}$$

where Γ and Δ are the vectors of modal coefficients.

$$\Gamma = [\gamma_1, \gamma_2, \ldots, \gamma_n] \qquad \Delta = [\delta_1, \delta_2, \ldots \delta_n] \tag{13}$$

and M, the mapping matrix, can be developed by a sequence of calculations that use modal projections for each level of the end- force as follows

$$\gamma_i(t) = \langle y_2(x,t), \phi_i(x) \rangle = \left\langle \sum_{j=2}^{N} \beta_j(t)\psi_j(x), \phi_i(x) \right\rangle = \sum_{j=2}^{N} \beta_j(t) \langle \psi_j(x), \phi_i(x) \rangle \tag{14}$$

and similarly

$$\beta_j(t) = \sum_{k=2}^{N} \delta_k(t) \langle \psi_j(x), \phi_k(x) \rangle^{1} \tag{15}$$

Now substituting $\beta_j(t)$ from Eq.(15) into Eq.(14), results in

$$\gamma_i(t) = \sum_{j=2}^{N} \sum_{k=2}^{N} \langle \phi_i(x), \psi_j(x) \rangle \langle \psi_j(x), \phi_k(x) \rangle \delta_k(t) \tag{16}$$

Comparing this equation with Eq.(12), reveals that the structure of the mapping matrix M. It should be noticed the convergence of this process depends on the $N \times N$ linear operator M, which can be constructed as follows: the first column contains all zeros since the first modal coefficient was zeroed out (note that this implies that M will always have at least one zero eigenvalue). The remaining columns are filled in by the coefficient

$$\langle \phi_i(x), \psi_j(x) \rangle \langle \psi_j(x), \phi_k(x) \rangle, \; i = 1,2,\ldots,N \quad k = 2,3,\ldots,N.$$

---

[1] $\langle f, g \rangle$ is the inner product of function f and g denoted by $\int_0^L fg\,dx$.

If all eigenvalues of M lie inside the unit circle, the process will converge, implying that all modes consideration die out under repeated cycling and removal of the first relevant mode. In fact rate of convergence (or divergence) is dictated by these eigenvalues.

An example using the cantilever beam for $p_1 = 0$ and $p_2 = 75N$, using $N = 3$ modes is derived to demonstrate the methodology. In this case the matrix for the map is developed using the results derived above (and Fig.3) and the resulted eigenvalues are shown below

$\lambda_1 = 0.0 \quad \lambda_2 = 0.502 \quad \lambda_3 = 0.903$

As it can be noticed the rate of convergence for higher modes is much smaller compared to the lower modes. In other words, absolute values of the eigenvalues that correspond to higher modes, are close to unity. Notice, in this calculation the effect of the structural damping is not taken into the consideration. In practice, structural damping accelerates the convergence rate of the higher modes (to zero) in the modal disparity approach.

Preliminary Experiments

A follower force is very difficult to implement in a cantilever beam. We therefore implemented an end force that closely resembles a follower force, as shown in Fig.4. A Kevlar cable is used to apply the end force in the beam. The attachment at the tip of the beam houses a pair of bearings; these bearings will allow the Kevlar cable to smoothly wrap around the front face. The cable tension mechanism, which consists of a solenoid and a load cell is also shown in the figure.

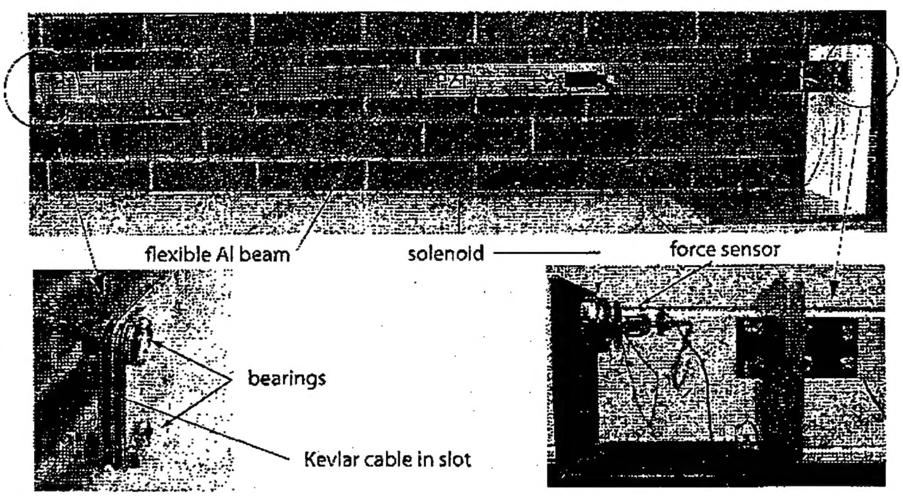

Figure 4 The cantilever beam and end force mechanism.

Due to the short duration of our project (8 months) we were unable to perform extensive experiments. Some preliminary experimental results are shown here that lend credibility to the concept of modal disparity. In Figs.5 (a) and (b), we show the vibration in the beam after application of an end force for the two cases where the beam was initially vibrating purely in the first mode and second mode, respectively. In both cases, it can be seen that

the beam has energy in both the first and second modes after application of the end force. This obviates that by removing the energy from the first mode only and switching between two different levels of the end force, it will be possible to suppress the vibration of the beam completely.

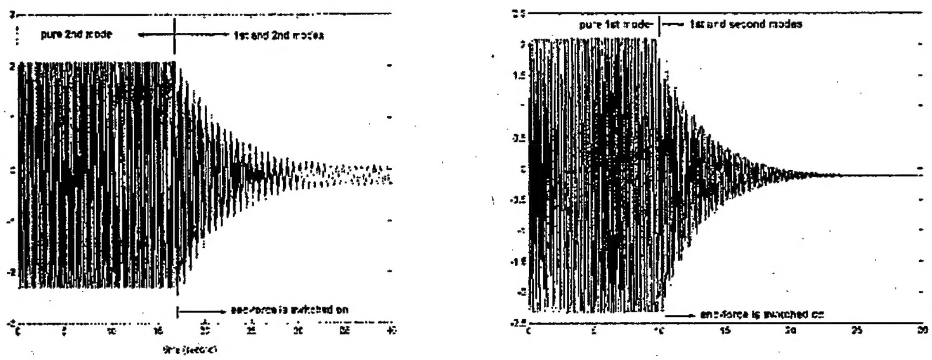

Fig.5. Demonstration of energy flow between modes due to modal disparity generated by switching of end forces.

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